Topic : Forms of Quadratic Functions

Goal : I know the three forms that a quadratic function can be written in and what information can be taken directly from the equation for each.

Forms of Quadratic Functions

Using technology, graph each of the following functions.
What do you notice?

They all represent the same parabola!

A. $y = x^2 + 2x - 3$

Standard Form
$y = ax^2+bx+c$

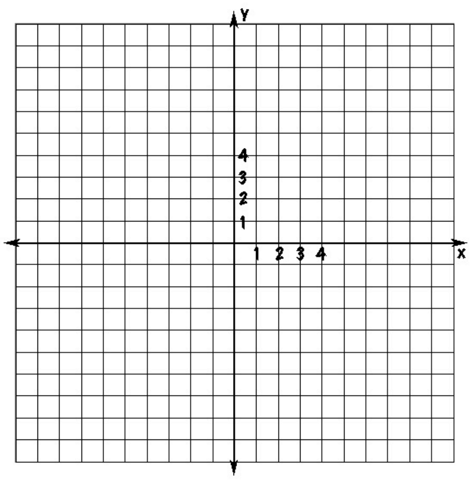
B. $y = (x + 3)(x - 1)$

Factored Form
$y = a(x - r)(x - s)$

C. $y = (x+1)^2 - 4$

Vertex Form
$y = a(x-h)^2+k$

Graph the parabola and state its properties.



Vertex : ______________________________

Opening : ______________________________

Axis of symmetry: ______________________________

Max/Min Value : ______________________________

x-intercepts : ______________________________

y-intercepts : ______________________________

Which properties is each form useful in finding?

Standard Form	**Factored Form**	**Vertex Form**
$y = ax^2+bx+c$	**$y = a(x - r)(x - s)$**	**$y = a(x-h)^2+k$**
$y = x^2 + 2x - 3$	$y = (x + 3)(x - 1)$	$y = (x+1)^2 - 4$

Example 1. For $y = -3x^2 + 4x - 7$ state...

a) the direction of opening ______________________

b) does it have a max or min? ______________________

c) the y-intercept? ______________________

$$y = -3x^2 + 4x - 7$$

Example 2. a) What are the x-intercepts of $f(x) = (x + 6)(x - 4)$?
b) Use the x-intercepts to locate the vertex.
c) What is the y-intercept?

$$y = (x + 6)(x - 4)$$

☆ *Because of symmetry, the vertex will be directly between the two intercepts. So take the average of the intercept points and you will find the x-coordinate of the vertex.*

☆ *If you know the x-coordinate of the vertex, the y-coordinate is simply the value of the function at that location, so determine f(-1).*

☆ *When the parabola goes across the y-axis, the x-value is ZERO. To find the y-intercept, let x=0.*

NOTE

The vertex really is the most important part of the parabola. Once you know it, you also know

*** axis of symmetry** *** max/min value** *** range**

Practice Questions - Handout Page

MBF3C U3L1 Forms of Quadratic Functions

1. The following equations are in STANDARD FORM. Identify the values of a, b and c for each.

a) $y = 2x^2 + 4x + 8$ a=_________ b=_________ c=_________

b) $y = x^2 + 9x - 6$ a=_________ b=_________ c=_________

c) $y = -x^2 - 2x$ a=_________ b=_________ c=_________

d) $y = \frac{1}{2}x^2 - 16$ a=_________ b=_________ c=_________

2. The following are the same equations from question #1. Circle the appropriate response or fill in the blank.

a) $y = 2x^2 + 4x + 8$ Opens : U or D Max or Min y-int : _________

b) $y = x^2 + 9x - 6$ Opens : U or D Max or Min y-int : _________

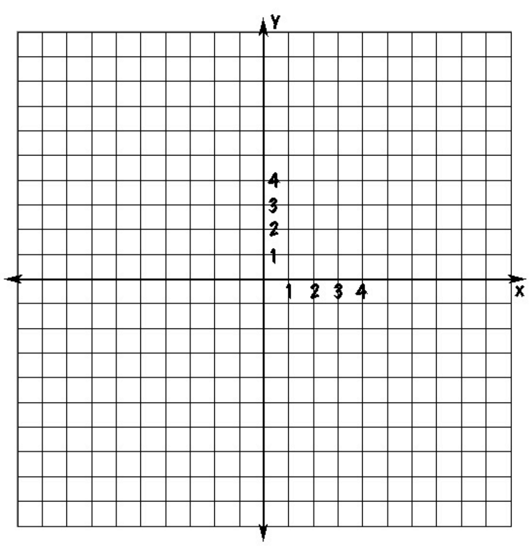
c) $y = -x^2 - 2x$ Opens : U or D Max or Min y-int : _________

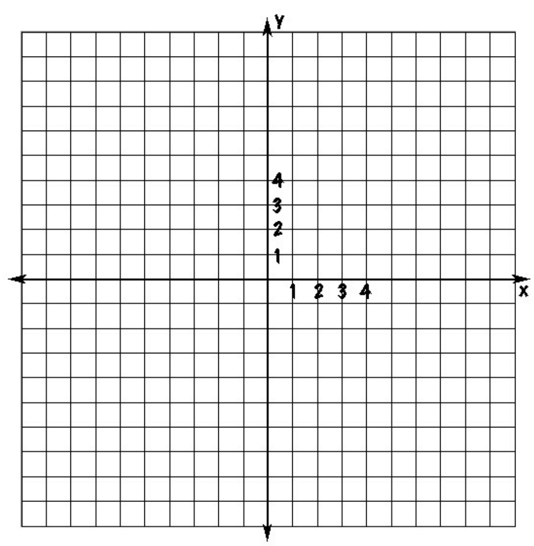
d) $y = \frac{1}{2}x^2 - 16$ Opens : U or D Max or Min y-int : _________

3. The following quadratic equations are in FACTORED FORM. Fill in the following chart. You will need to do some work on a scrap piece of paper…

	Direction of Opening	*x-intercepts*		*Coordinates of the vertex*	*Max/Min value*	*y-intercept*
a) $y = (x-3)(x-7)$				(,)	Max Min	
b) $y = -(x+1)(x+9)$				(,)	Max Min	
c) $y = -2(x-4)(x+2)$				(,)	Max Min	
d) $y = \frac{1}{2}x(x+6)$				(,)	Max Min	

4. Use your information from #3 to graph the ABOVE equations on the grids below.

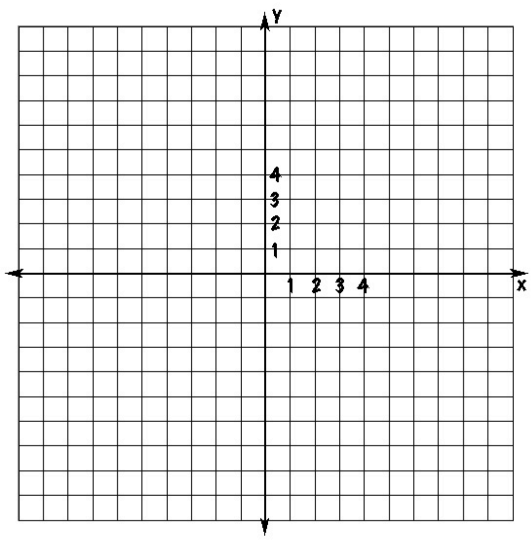


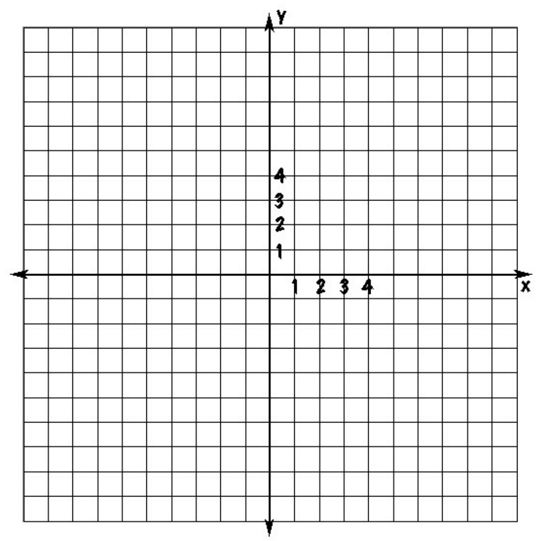


5. The following functions are in VERTEX FORM. Fill in the chart

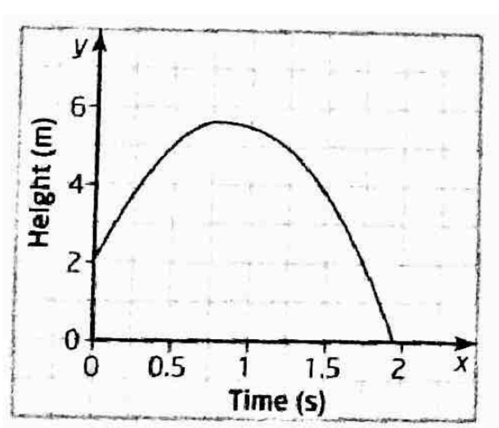
	a	*h*	*k*	*Vertex*	*Axis of Symmetry*	*Maximum or Minimum Value*	*y-intercept*
a) $y = (x-3)^2 + 2$							
b) $y = (x+4)^2 - 5$							
c) $y = -3(x+1)^2 - 6$							
d) $y = \frac{1}{2}(x-2)^2 - 8$							

6. Use your information from #5 to graph the ABOVE equations on the grids below.





7. The given graph shows the height-time relationship of a bottle rocket.



a) From what height above the ground was this bottle rocket launched?

b) What is the approximate maximum height reached?

c) Estimate the “hang time” of the rocket.

d

d) Extend the function backwards into negative time. What is the other x-intercept?
e) Use the x-intercepts to find the vertex. What is the equation of the graph in vertex form.
f) Suppose the launching platform were lowered. How would this effect the y-intercept, the vertex, and the x-intercept of the new graph and its hang-time.

Topic : Switching to Standard Form

Goal : I can expand and simplify expressions and change quadratic functions into standard form.

Switching to Standard Form

Let's quickly recap how to expand and simplify...

Example 1. Expand and simplify each of the following polynomials.

a) $(2x + 5)(3x - 7)$

☆ Use Double Distributive law (FOIL) to expand the brackets

b) $7(p - 4)(8 - p)$

☆ First expand and simplify the brackets, then multiply the coefficient through.

c) $(5t - 3)^2$

☆ Remember that when a bracket is squared, that means we really have two of the same brackets multiplied together.

Switching from Factored form to Standard Form

Write $y = 4(x + 3)(x + 5)$ in standard form.

Basically all I'm asking you to do is to expand and simplify the expression on the right.

$y = 4(x + 3)(x + 5)$

☆ First expand and simplify the brackets, then multiply the coefficient through.

Switching from Vertex form to Standard Form

Write $y = 4(x - 3)^2 - 7$ in standard form.

Basically all I'm asking you to do is to expand and simplify the expression on the right.

- ☆ Write the squared bracket as two brackets multiplied together
- ☆ Expand the two brackets.
- ☆ Multiply the coefficient through the brackets.
- ☆ Simplify and be sure it is in order of descending powers of x.

Example 2. Put the following parabolas in standard form.

a) $y = 4(x - 6)(x + 3)$

b) $y = 2(x-6)^2 - 8$

Example 3. A word problem.

A study shows that 60,000 students will attend a play in one week if the ticket price is 40 dollars. For every 2.50 dollars added to the ticket price, 2000 fewer students will attend the play.

The amount of money the theatre will earn (before expenses) is called revenue. Revenue = (# tickets sold)(cost per ticket)

a) What is the revenue when the tickets are $40?

b) The Revenue can be written as a function of "n" where n is the number of times the price increases by $2.50.

$$R = (60\ 000 - 2000n)(40 + 2.5n)$$

Explain how that equation was found

c) Write the equation in standard form. Graph using technology. What is the maximum revenue and what ticket price will give it?

Practice Questions - Handout Page

MBF3C U3L2ws Forms of Quadratic Functions

1. a) State the x-intercepts for each of the following functions, and use them to find the vertex.

a) $y=(x-3)(x+8)$

x-int: ________ and _______

b) $y=(x-2)(x-6)$

x-int: ________ and _______

c) $y=-(x+1)(x-9)$

x-int: ________ and _______

b) Expand and simplify each equation above so that it is in STANDARD FORM. What is the y-intercept?

a)

y-int: _________

b)

y-int: _________

c)

y-int: _________

c) Use the 5 points you have for each parabola to sketch a graph on a separate piece of graph paper.

2. Complete all the same steps from question #1 for the following parabolas.

a) $y=3(x+2)(x-5)$ b) $y=-4(x+2)^2$ c) $y=\frac{1}{2}(x+6)(x-4)$

3. Express each quadratic function in standard form and identify the y-intercept. Sketch a graph of the function.

a) $y=(x+2)^2+3$ b) $y=3(x-5)^2+8$ c) $y=-4(x+3)^2-9$

4. Expand and simplify using tools and methods of your choice.

a) $2(x-3)+3(x-6)$ b) $y=(y+4)(y-4)-(y-3)(y-4)$ c) $3(p-1)^2-2(p+3)(p-4)$

Topic : Factoring Simple Trinomials

Goal : I know how to factor simple trinomials so that I can find the x-intercepts of quadratic functions (parabolas)

Factoring Simple Trinomials

We saw that when we have a function in factored form, we can easily pick out the x-intercepts.

$$y=(x+3)(x-4)$$

So if we want the x-intercepts, it would be nice if we could put the equation into factored form. In this lesson we discuss how to put a polynomial into factored form.

We will start by expanding and looking for patterns. We can put all expansion questions into two categories...

Brackets have the same signs...

(x+2)(x+5) =

(x-2)(x-5) =

So if the brackets have the same signs, we notice that in the trinomial answer...

1) Has a constant term that is always _____________.

2) Has a middle term that has the same sign as

3) Has a constant term from ______________ the constants in the brackets.

4) Has a middle term that comes from _____________ the constants in the brackets.

Brackets have different signs...

(x+2)(x-5) =

(x-2)(x+5) =

So if the brackets have different signs, we notice that in the trinomial answer...

1) Has a constant term that is always _____________.

2) Has a middle term that has the same sign as

3) Has a constant term from ______________ the constants in the brackets.

4) Has a middle term that comes from _____________ the constants in the brackets.

Let's use these patterns to write the two brackets that these trinomial answers come from...

$y = x^2 - 9x - 36$

$y = x^2 + 10x + 24$

$y = x^2 + 5x - 24$

$y = x^2 - 7x + 12$

Steps to factoring.

Step 1. Put down two sets of brackets and place an x at the front of each.

Step 2. Look at the sign of the constant term.

Step 3. If the constant term is +

The brackets have the same sign and it will be the same as the sign of the middle term. So put that sign in both brackets.

Step 4. Since the constant term is + we are looking for two numbers that multiply to the constant term and + to the middle term.

Step 3. If the constant term is -

The brackets have different signs, so put one of each signs in the brackets.

Step 4. Since the constant term is - we are looking for two numbers that multiply to the constant term and - to the middle term.

Step 5. Put the bigger of the two numbers you find in the bracket with the same sign as the middle term.

More examples. Factor the following...

$n^2 + 10n + 16$

$r^2 + r - 20$

$m^2 + m - 6$

$b^2 - 9b + 14$

$p^2 - 8p + 7$

$b^2 - 8b + 15$

$k^2 - 4k - 60$

$p^2 - 2p - 15$

Practice Questions - Handout Page

MBF3C U3L3ws Factoring Simple Trinomials

1. $x^2 + 9x + 14$
2. $x^2 + 12x + 35$
3. $x^2 + 20x + 36$
4. $x^2 + 15x + 50$
5. $x^2 - 20x + 75$
6. $x^2 - 33x + 32$
7. $x^2 + 17x + 42$
8. $x^2 - 20x + 36$
9. $x^2 - 17x + 16$
10. $x^2 - 1$
11. $x^2 + 16x + 48$
12. $x^2 + 26x + 48$
13. $x^2 + 15x + 44$
14. $x^2 - 10x - 24$
15. $x^2 - 18x + 32$
16. $x^2 + 4x - 12$
17. $x^2 + 10x - 24$
18. $x^2 -14x -32$
19. $x^2 - 4$
20. $x^2 + 28x + 75$
21. $x^2 + 20x + 75$
22. $x^2 - 14x + 48$
23. $x^2 - 23x + 42$
24. $x^2 - 14x + 33$
25. $x^2 - x - 12$
26. $x^2 - 6x - 16$
27. $x^2 - 4x - 32$
28. $x^2 - 13x + 36$
29. $x^2 - 9$
30. $x^2 - 12x + 32$
31. $x^2 + 17x + 70$
32. $x^2 -15x - 16$
33. $x^2 +10x - 24$
34. $x^2 - 16x - 36$
35. $x^2 - 6x - 40$
36. $x^2 - 10x + 24$
37. $x^2 - 8x + 16$
38. $x^2 - 5x - 36$
39. $x^2 - 25$
40. $x^2 - 14x + 24$
41. $x^2 - 27x + 50$
42. $x^2 - 19x + 48$
43. $x^2 - 18x + 45$
44. $x^2 + 15x + 54$
45. $x^2 - 5x - 24$
46. $x^2 - 16$
47. $x^2 - 12x + 32$
48. $x^2 - 33x + 32$
49. $x^2 - 36$
50. $x^2 - 10x + 25$
51. $x^2 + 19x + 60$
52. $x^2 - 24x + 44$
53. $x^2 - 21x + 54$
54. $x^2 + 11x - 12$
55. $x^2 - 23x - 24$
56. $x^2 - 10x - 24$
57. $x^2 + 12x + 36$
58. $x^2 + 16x - 36$
59. $x^2 - 49$
60. $x^2 - 12x + 36$
61. $x^2 - 16x + 60$
62. $x^2 + 23x - 24$
63. $x^2 + 4x - 32$
64. $x^2 - 3x - 40$
65. $x^2 - 13x + 40$
66. $x^2 - 64$
67. $x^2 + 13x + 36$
68. $x^2 + 18x + 32$
69. $x^2 + 25x + 24$
70. $x^2 - 37x + 36$
71. $x^2 + 14x + 45$
72. $x^2 + 46x + 45$
73. $x^2 + 15x - 16$
74. $x^2 + 6x -16$
75. $x^2 - 81$
76. $x^2 - 23x + 60$
77. $x^2 - 19x + 70$
78. $x^2 - 3x - 4$
79. $x^2 - 10x - 16$
80. $x^2 - 31x - 32$

MBF3C U3L3ws Factoring Simple Trinomials

1. $x^2 + 9x + 14$
2. $x^2 + 12x + 35$
3. $x^2 + 20x + 36$
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5. $x^2 - 20x + 75$
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28. $x^2 - 13x + 36$
29. $x^2 - 9$
30. $x^2 - 12x + 32$
31. $x^2 + 17x + 70$
32. $x^2 -15x - 16$
33. $x^2 +10x - 24$
34. $x^2 - 16x - 36$
35. $x^2 - 6x - 40$
36. $x^2 - 10x + 24$
37. $x^2 - 8x + 16$
38. $x^2 - 5x - 36$
39. $x^2 - 25$
40. $x^2 - 14x + 24$
41. $x^2 - 27x + 50$
42. $x^2 - 19x + 48$
43. $x^2 - 18x + 45$
44. $x^2 + 15x + 54$
45. $x^2 - 5x - 24$
46. $x^2 - 16$
47. $x^2 - 12x + 32$
48. $x^2 - 33x + 32$
49. $x^2 - 36$
50. $x^2 - 10x + 25$
51. $x^2 + 19x + 60$
52. $x^2 - 24x + 44$
53. $x^2 - 21x + 54$
54. $x^2 + 11x - 12$
55. $x^2 - 23x - 24$
56. $x^2 - 10x - 24$
57. $x^2 + 12x + 36$
58. $x^2 + 16x - 36$
59. $x^2 - 49$
60. $x^2 - 12x + 36$
61. $x^2 - 16x + 60$
62. $x^2 + 23x - 24$
63. $x^2 + 4x - 32$
64. $x^2 - 3x - 40$
65. $x^2 - 13x + 40$
66. $x^2 - 64$
67. $x^2 + 13x + 36$
68. $x^2 + 18x + 32$
69. $x^2 + 25x + 24$
70. $x^2 - 37x + 36$
71. $x^2 + 14x + 45$
72. $x^2 + 46x + 45$
73. $x^2 + 15x - 16$
74. $x^2 + 6x -16$
75. $x^2 - 81$
76. $x^2 - 23x + 60$
77. $x^2 - 19x + 70$
78. $x^2 - 3x - 4$
79. $x^2 - 10x - 16$
80. $x^2 - 31x - 32$

Topic : Simple Trinomial Factoring - Special Cases

Goal : I know how to factor some quadratics that have common factors or are missing terms.

Simple Trinomial Factoring - Special Cases

Special Case #1 - there is a common factor

Sometimes a simple trinomial can be disguises as a complex one by a common factor.

$$3x^2 + 3x - 18$$

At first it looks complicated, but once you realize that you can take out a common factor of 3, it's really a very simple trinomial to factor.

$$3x^2 + 3x - 18$$

So, if you see a number in front of the x^2-term, you will likely be able to divide every term in the trinomial by that number. Then you can just ignore it, and factor as usual.

Special Case #2 - there is no x-term

$$5x^2 - 45$$

First of all you want to remove the common factor. The number in front of x-squared can be divided out of both terms.

Then just know that the middle term missing means that the coefficient of the x-term must have been zero.

Special Case #3 - there is no constant term

$$y=4x^2+20x$$

First take out the common factor. But this time since the constant term is missing, you can also divide out an x along with it.

What are the x-intercepts of this quadratic function?

Examples. Factor each of the following quadratic expressions.

a) $3x^2 + 24x + 36$

b) $0.5x^2 - 5x + 8$

c) $2x^2 - 10x - 48$

d) $4x^2 - 100$

c) $6x^2 - 30x$

Practice Questions - Handout Page

MBF3C U3L4ws

Simple Trinomial Factoring - Special Cases

Factor each completely.

1) $6x^2 + 12x - 378$

2) $2b^2 - 38b + 180$

3) $2x^2 - 14x + 12$

4) $2a^2 - 14a + 20$

5) $6x^2 - 42x - 48$

6) $-6a^2 - 30a - 24$

7) $-6n^2 - 36n$

8) $2r^2 - 32r + 120$

9) $5n^2 - 15n - 140$

10) $-4x^2 - 8x$

11) $3a^2 - 39a + 108$

12) $4n^2 + 32n + 48$

13) $4v^2 + 32v - 36$

14) $6v^2 + 54v + 108$

15) $3m^2 - 3m - 6$

16) $3x^2 - 3x$

17) $5x^2 - 320$

18) $-4x^2 - 44x - 40$

19) $-4b^2 - 48b - 140$

20) $-3x^2 + 18x$

21) $3b^2 - 30b$

22) $2x^2 + 22x + 48$

23) $6x^2 - 78x + 252$

24) $2p^2 - 14p - 60$

25) $-3a^2 - 6a$

Answers to Simple Trinomial Factoring - Special Cases

1) $6(x + 9)(x - 7)$
2) $2(b - 9)(b - 10)$
3) $2(x - 6)(x - 1)$
4) $2(a - 2)(a - 5)$
5) $6(x + 1)(x - 8)$
6) $-6(a + 4)(a + 1)$
7) $-6n(n + 6)$
8) $2(r - 10)(r - 6)$
9) $5(n - 7)(n + 4)$
10) $-4x(x + 2)$
11) $3(a - 4)(a - 9)$
12) $4(n + 6)(n + 2)$
13) $4(v + 9)(v - 1)$
14) $6(v + 3)(v + 6)$
15) $3(m - 2)(m + 1)$
16) $3x(x - 1)$
17) $5(x - 8)(x + 8)$
18) $-4(x + 1)(x + 10)$
19) $-4(b + 5)(b + 7)$
20) $-3x(x - 6)$
21) $3b(b - 10)$
22) $2(x + 8)(x + 3)$
23) $6(x - 6)(x - 7)$
24) $2(p + 3)(p - 10)$
25) $-3a(a + 2)$

Topic : The Factored Form of a Quadratic Relation

Goal : I know how to graph quadratic equations from the factored form and how to get information from the quadratic form of an equation.

The Factored Form of a Quadratic Relation

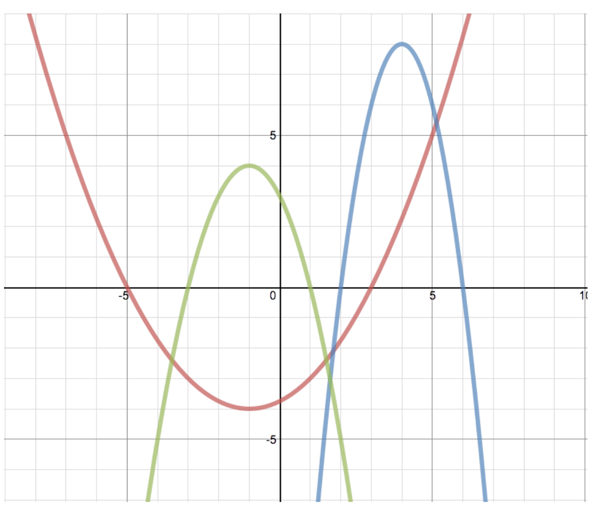
What kind of things might you need to do with quadratic equations in factored form? Here are a few examples.

Example 1. Write the following in factored form. What are the x-intercepts?

a) $y = x^2 - 4x - 21$

b) $y = -2(x-1)^2+8$

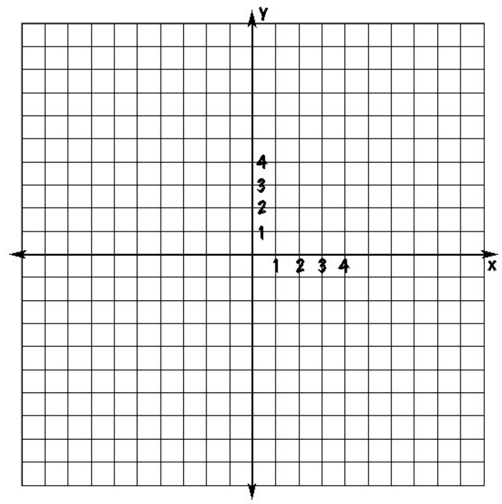
Example 2. Use the graph of the following parabolas to write each equation in both vertex and factored form.



Example 3. Chose one of the graphs in example two and expand both equations in standard form. They should both be the same.

Example 4. Find the vertex of the given equation and graph it on the grid provided.

$$y = -\frac{1}{2}(x-3)(x-7)$$



x-intercepts: ______________

Finding the x-coordinate of the vertex.

Finding the y-coordinate of the vertex.

Now graph using the 3 points you have found - you can find more by using the a-value and the vertex point.

Practice Questions - Page 141 #1-6, 9